Forecasting the term-structure of euro area swap rates and Austrian yields based on a dynamic Nelson-Siegel approach

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September 11, 2018

Abstract

We employ an extended Nelson-Siegel model to produce a macro-financial framework for forecasting the euro area term-structure of interest rates and the Austrian yield curve, with a view to enabling risk management analyses and determining an optimal debt management strategy. We determine the dynamics of the term-structure by the movements of the level, slope and curvature parameters, influenced by the cyclical position of the economy and the price level. Long-term interest rates are based on the assumption that the level, slope and curvature of the yield curve and macroeconomic variables converge to their historic mean. Using in-sample and out-of-sample forecast benchmarking, we find that our model clearly outperforms the expectations hypothesis of the term-structure of the interest rates (forward rate) forecast, while a constant expectations (random walk) forecast appears to produce a superior out-of-sample performance, partly due to the existence of different policy regimes in our data sample. We then use our model to forecast the term-structure of euro area interest rates and the Austrian yield curve until 2028.

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Disclaimer: This paper does not necessarily express the views of the Austrian Fiscal Advisory Council or the Austrian Treasury.

Acknowledgements: We would like to thank Kurt Hornik, Bernhard Knapp, Kevin Kurt, Stefan Pichler and Thomas Steiner for their useful comments. All remaining mistakes are ours.

1 Introduction

This study develops a term-structure model for euro area interest rates and Austrian yields based on financial and macroeconomic variables. The main purpose of this model is to forecast the term-structure of interest rates for the euro area and the Austrian yield curve, which the Austrian debt management office (OeBFA) needs to perform risk management analyses and determine an optimal debt management strategy.

The Term-structure of interest rate models essentially consist of three main elements: the selected state variables describing interest rates, their dynamics and interaction, and their mapping to the term-structure. The wide range of literature on term-structure modeling offers a variety of potential assumptions and combinations concerning these three elements.Based on the promising documented results of out-of-sample forecasting performance, we follow Diebold et al. (2006) and employ a dynamic Nelson-Siegel (NS) term-structure model with three latent state variables that can be interpreted as the level, slope and curvature of the yield curve. With respect to mapping the state variables to the term structure, the three-factor NS model deviates from the no-arbitrage condition of the affine term structure models used widely throughout the body of literature (Duffie and Kan, 1996,Dai and Singleton, 2000, Duffie et al., 2003 and Ang et al., 2007). The reason for doing so lies in their identified relatively weak forecasting performance (e.g. Bolder and Deeley (2011)). Following Diebold et al. (2006) we assume that the three parameters are influenced by macroeconomic variables. In order to capture the dynamic behavior and interaction of the state and macroeconomic variables we employ a multi-country (euro area and Austria) vector autoregressive (VAR) model. The macroeconomic variables are modeled to have only an indirect impact on the term-structure via their influence on the dynamics of the NS parameters (factor loadings), and not a direct impact as state variables.

There are two benefits of including macroeconomic variables in our model. First, as the body of literature shows, the dynamics of macroeconomic variables provide useful information that can improve forecasting performance (e.g. Bolder and Liu, 2007). Second, by jointly evaluating macroeconomic variables and yields, we can assess the potential budgetary risk stemming from macroeconomic dynamics via changes in interest payments, as discussed in the context of public debt management by Lloyd-Ellis and Zhu (2001) and Faraglia et al. (2008).[1] In a nutshell, this literature uses the joint dynamics of interest rates and macroeconomic variables to compensate for budgetary movements caused by macroeconomic dynamics by changed interest payments.

After constructing the term-structure model for euro area swap rates and the Austrian yields we perform pseudo out-of-sample forecasts to evaluate the model's forecasting ability. In line with the literature on forecast evaluation, we calculate mean absolute errors (MAE) and

[1] An application for Austria can be found in Fenz and Holler (2017).

root mean square errors (RMSE) to evaluate the forecasting performance of our model. In addition our model is benchmarked against forecasts derived from forward rates (rational expectation hypothesis) and a random walk process (constant expectations). Finally, we derive a forecast of the term structure of euro area interest rates and the Austrian yield curve for 2018 to 2028. By jointly analyzing euro area swap rates and Austrian yields we are further able to derive forecasts for country specific spreads for the Austrian yield curve.

2 Model

Our forecasting model can be split into two main parts: a three-factor model that describes the term structure of interest rates for the euro area (EA) and Austria (AT) and a multi-country VAR model that determines the dynamics and interaction of the factors (NS parameters) and macroeconomic developments. Considering the choice of state variables and the mapping to the term structure our model is based on extensions of the original NS model (Nelson and Siegel, 1987) by Diebold and Yue (2008) and Diebold et al. (2006). When modeling the dynamics of the state variables, we substantially deviate from the existing body of literature by employing a multi-country VAR model, since our model specifically aims to derive forecasts for the euro area and Austria, a euro area member state.

2.1 The dynamic Nelson-Siegel model

The original NS model uses a continuous functional form of forward interest rates to describe the yield curve. To make our model framework easier to understand, we choose to use the original model formulated in terms of zero-coupon yields:

$$y_t(\tau) = \beta_{0_t} + \beta_{1_t}\frac{1-e^{-\lambda\tau}}{\lambda\tau} - \beta_{2_t}e^{-\lambda\tau}, \tag{1}$$

where y is the continuously compounded zero-coupon yield to maturity for tenor τ. An alternative factorization and dynamic extension of the model by Diebold et al. (2006) allows the three NS parameters β_1, β_2 and β_3 to vary over time and enables an economic interpretation of the three factors as the level, slope and curvature of the yield curve. Due to the similar monotonically decreasing shape of $\frac{1-e^{-\lambda\tau}}{\lambda\tau}$ and $e^{-\lambda\tau}$ in the original NS model the interpretation of the slope and curvature parameters is omitted and the factor loadings have to be very similar. In addition the high coherence in the two factors raises the issue of multicollinearity which implies substantial problems to obtain a precise estimate of the corresponding factor loadings. With this insights in mind, we decide to follow this dynamic NS approach which is described by:

$$y_t(\tau) = \beta_{0_t} + \beta_{1_t} \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - \beta_{2_t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda_t\tau} \right) \quad (2)$$

or

$$y_t(\tau) = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}' \begin{bmatrix} 1 \\ \frac{1-e^{-\lambda\tau}}{\lambda\tau} \\ \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda_t\tau} \right) \end{bmatrix} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}' \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix} \quad (3)$$

The factor loadings $\beta_i, i = 0, 1, 2$ crucially determine the shape of the term structure of interest rates. Parameter β_0 corresponds to the intercept, β_2 steers the slope and β_3 steers the curvature of the yield curve. Changes in β_0 create parallel/level shifts in the yield curve. A higher β_1 increases the slope of the yield curve, while a lower value flattens the curve (assuming that the yield curve is upward sloping). A higher β_2 increases the curvature of the yield curve while negative values decrease the curvature. The parameter λ governs the exponential decay rate and changes the shape of the yield curve. If we observe the limiting behavior with respect to time to maturity the slope and curvature components vanish and the yield converges to the constant level of interest rate β_0. Economically β_0 can therefore be interpreted as the interest rate for very long-maturities. With respect to λ, a small value produces a slow rate of decay and better fits the curve for long maturities, while large values produce a fast rate of decay and better fit the curve for short maturities. The decay rate also governs the point at which β_2 achieves its maximum value.

2.2 The extended Nelson-Siegel model: the dynamics of the state variables

To evaluate the dynamics and interaction of the level-, slope- and curvature parameters we employ a VAR model. The VAR model implicitly assumes that the three parameters and their lagged values interact with each other. In addition, in line with Bolder and Liu (2007) we also assume that the factor loadings of these inherently latent variables are also influenced by macroeconomic dynamics. In this respect, we draw on influential papers by Ang and Piazzesi, 2003 Ang et al. (2007), Rudebusch and Wu (2008), Hoerdahl et al. (2006), Diebold et al. (2006), and **?**, which argue that macroeconomic variables are useful in estimating yield curve dynamics. Based on the main macroeconomic drivers influencing the yield curve as identified by these works, we consider the price level of the economy and its cyclical position to be the driving macroeconomic variables behind yield curve movements. The ratio behind this choice clearly reflects the the decision to take into account some kind of Taylor rule, which contrary to its conventional versions (e.g. Taylor, 1993 and Clarida et al., 1998) explains not only the short-term interest rates but the whole yield curve. The VAR(p) model is described by

$$\beta_{i_t} = c + B_p \beta_{i_{t-1}} + \epsilon_t, \tag{4}$$

where β_i is a $k \times 1$ vector containing the level, slope and curvature parameters and the macroeconomic variables. The $k \times k$ matrix B_p describes the dynamic development and interaction for the NS parameters and macroeconomic variables. p determines the lag structure of the VAR.

At this point we would like to remind the reader, that in our framework the macroeconomic variables do not influence the interest rates directly. Instead, they influence the dynamics of the NS parameters β_0, slope β_1 and curvature β_2. This indirect modeling approach has the advantage that the amount of parameters evaluated by the VAR model is much smaller.[2]

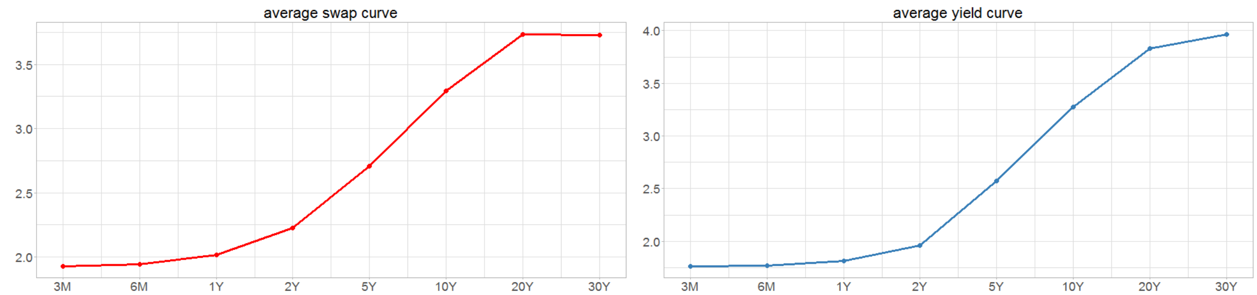
3 Data and estimation procedure

To derive the zero-compounded interest rates for the euro area we use linear interpolated benchmark end-of-month swap rates (bid-ask average) from January 1999 to January 2018 and apply a bootstrapping procedure to generate zero-compounded swap rates (y_t^{EA}). Our analysis considers eight different maturities $\tau = 3, 6, 12, 24, 60, 120, 240$ and 360 months. The raw interest rate data is taken from the Thomson Reuters OTC database. The left-hand graph in figure 1 plots the historical average of euro area interest rates (y_t^{EA}) for maturities τ.

The zero-compounded yield rates for Austria (y_t^{AT}) are also generated from linear interpolated benchmark end-of-month OTC rates (bid-ask averages) from January 1999 to January 2018, taken from Thomson Reuters via a bootstrapping procedure. We again consider 13 different maturities, for which five tenors were truncated from the analysis due to liquidity. Our analysis therefore took into account tenors $\tau = 3, 6, 12, 24, 60, 120, 240$ and 360 months. The historical average Austrian yield curve for our given time frames is presented in the right-hand graph in figure 1.

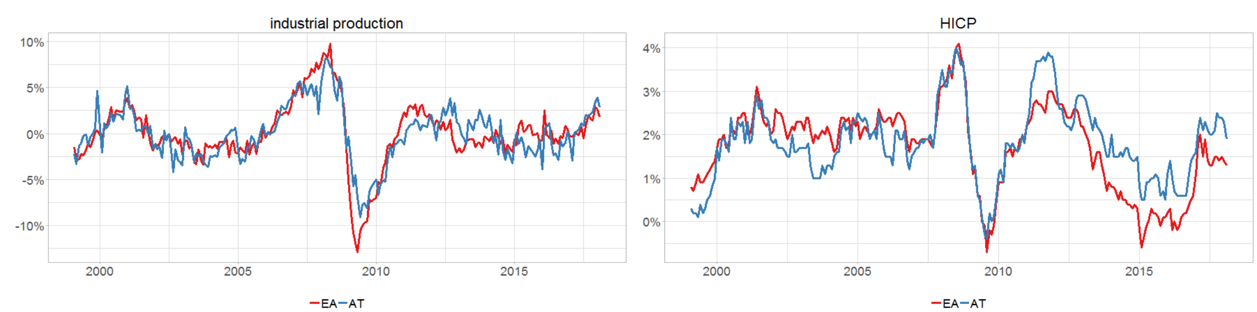
[2] Other term structure models directly model the link between interest rates and macroeconomic variables, e.g. Ang and Piazzesi, 2003.

Figure 1: EA swap rates and AT-yields



Drawing on a vast set of potentially viable macroeconomic data describing the dynamics of prices and the position of the economy in the economic cycle, we decide to use euro area and Austrian industrial production data provided by the OECD and HICP 12-month averages published by Eurostat (Π^{EA}, Π^{AT}). This was predominantly due to the frequency of the data used to determine the cyclical position and it being the most relevant observation of inflation in the context of the primary euro area monetary policy target. Commonly used output gap measures and underlying data series (e.g output gap published by the European Commission and the IMF) are only available on a quarterly basis. The industrial production data series by the OECD is the only series available on a monthly basis. To deduce the cyclical position of the economy we simply apply a HP-filter to the industrial production data.[3] The calculated cyclical deviations from trend industrial production are interpreted as the output gap of the economy (og^{EA}, og^{AT}) see left-hand graph in figure 2. HICP developments are plotted on the right-hand graph in figure 2.

Figure 2: Macroeconomic variables



[3] The employed smoothing parameter for monthly data is taken from Ravn and Uhlig (2002) and corresponds to 129600.

To avoid potentially challenging numerical optimizations (λ_t enters equation (2) in a nonlinear way) Diebold and Li (2006) suggest fixing parameter λ_t over time. Since this approach results in only a marginal loss of fit we fix λ_t, substantially simplifying the estimation procedure. This enables us to estimate the parameters β_1 by ordinary least square regression. To determine the fixed value of λ we identify the curvature of equation (2) maximizing values for given τ, $\tau \in \{0, 0.01, ..., 9.99, 10\}$. The obtained results for λ are then used to fit the NS equation to the data by means of ordinary least square estimation, which allows us to identify the RMSE-minimizing λ. The results of this analysis are shown in Table (1). Our RMSE-minimizing λ corresponds to tenor 3.91 and 3.98 years for swaps and yields respectively.

Table 1: Nelson-Siegel decay parameter

	EA Swap	AT Yield
λ	0.46	0.45
τ	3.91	3.98
Mean RMSE (in bp)	5.94	6.64

Now we turn our attention towards the modeling of the dynamics of the NS parameters. The VAR model describing the dynamics of the NS parameters of interest rates (y_t^j) where $j = EA, AT$ is represented by equation (4). To capture the long-term dynamics of (y_t^j) we decide to anchor the long-term values of β_i at historical means (μ_i). In other words we assume that the NS parameters and the macroeconomic variables converge to their long-term historical mean. To do so we describe and evaluate the VAR in mean-adjusted form

$$(\beta_{i_t}^j - \mu_i) = B_p(\beta_{i_{t-1}} - \mu_i) + \epsilon_t, \tag{5}$$

for $j = EA, AT$, where $\beta_i = ({\beta_0}^{AT}, {\beta_1}^{AT}, {\beta_2}^{AT}, {\beta_0}^{EA}, {\beta_1}^{EA}, {\beta_2}^{EA}, og^{AT}, \pi^{AT}, og^{EA}, \pi^{EA})'$. Information criteria and out-of-sample performance forms the basisi of our decision to set the lag parameter p equal to 1.[4]

3.1 Fitting the term-structure of interest rates

As a starting point for evaluating the performance of our model, this section examines whether the dynamic NS model employed represented by equation (2) is capable of reproducing the

[4] β_i were also checked for stationarity: all series with the exception of β_1 and β_2 are stationary at least at the 10% level. Since the eigenvalues of the VAR polynomial point towards a potential non-stationarity of the VAR model we considered an alternative specification as a VECM, which was rejected since variables are not collinear. We therefore retain the original formulation of the model.

actual observed behavior of the term-structure of interest rates. In general, the following stylized facts apply to the term-structure of interest rates[5]:

1. The average term-structure of interest rates, which corresponds to the yield curve with the average values for β_i, increases and is concave (figure 1).

2. The term-structure of interest rates shows a variety of shapes (upward sloping, downward sloping, humped, and inverted humped) over time.

3. The term-structure of interest rates is persistent over time. In addition, the short end of the curve is less persistent and more volatile than the long end.

4. The slope of the yield curve is more volatile than yields.

As discussed in Diebold et al. (2006), theoretically speaking our model is perfectly able to reproduce these stylized facts. We now focus on whether our estimated curves are able to empirically reproduce the curves actually observed. To determine this, we start by comparing our estimated NS parameters with the dynamics of historically observed euro area swap rates and Austrian yield curves (see figures 3 and 4). We find that the level parameter β_0 decreases over time, nicely capturing the observed level decrease for swap rates and yields throughout over the considered time horizon. Moreover, the figures show that periods with flat curves correspond to periods with low estimated negative slope parameters (β_1) while periods with steep curves correspond to periods with high estimated negative slope parameters. The link between the curvature parameter (β_2) and the behavior of the observed curves is less obvious and hard to evaluate from a graph. To analytically evaluate the correlation between our estimated parameters and the observed curve dynamics, following Diebold and Li (2006) we define empirical versions of the level $\hat{\beta}_0$, slope $\hat{\beta}_1$ and curvature $\hat{\beta}_2$ of yield curves to compare them with our estimated NS parameters.[6] The empirically determined parameters are highly correlated with our estimated NS parameters.

To check the overall performance of our model in terms of replicating the aforementioned stylized facts, we calculate the estimated (fitted) average rate curve and examine the ability of our framework to replicate different shapes of the term-structure of interest rates. Figure 5 shows that the NS parameters are capable of replicating the (actual) average swap rate and yield curve. The fact put forward in Diebold and Li (2006), that the model has difficulties when the yields are dispersed and have multiple interior minima and maxima is also true

[5]The following list is based on descriptions by Diebold et al. (2006).

[6]The empirical level $\hat{\beta}_0$ is defined as the 10-year swap rate. The empirical slope $\hat{\beta}_1$ is defined as the difference between 10-year and 3-month swap rates, and the empirical curvature $\hat{\beta}_2$ as twice the 2-year swap rate minus the sum of the 3-month and 10-year swap rates.

for our framework. We can further observe that spreads are more volatile than yields. This allows us to conclude, that our model captures all of the above mentioned stylized facts and that the fit of our model is highly satisfactory.

Figure 3: Nelson-Siegel parameters EA swap rates

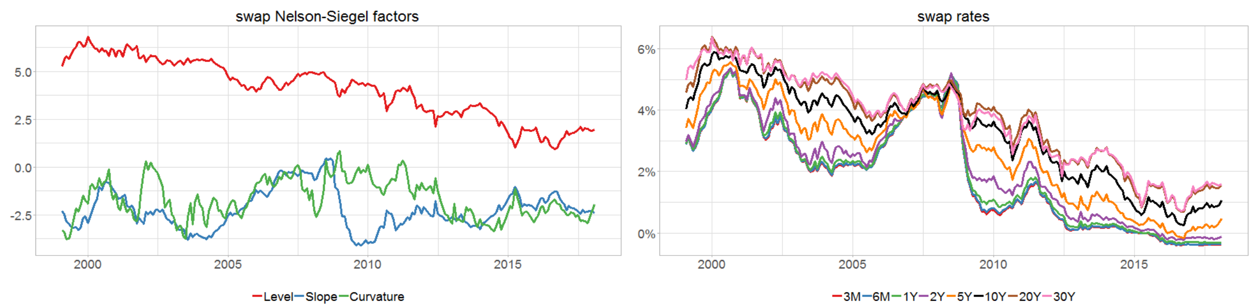


Figure 4: Nelson-Siegel parameters AT yield rates

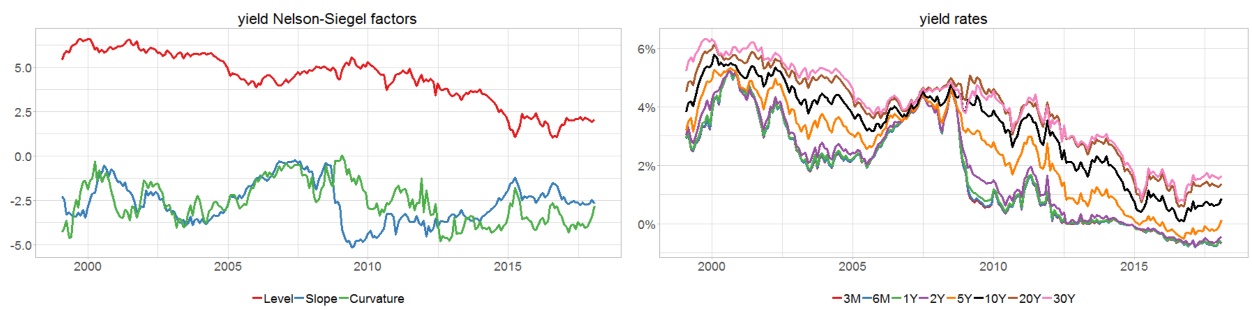
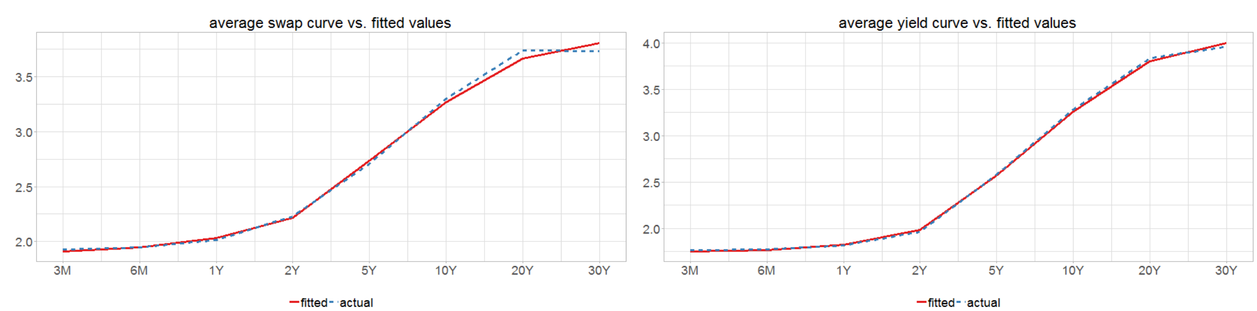


Figure 5: Fitting the model to actual data



4 Forecast evaluation

This section focuses on our model's ability to forecast historically observed term-structures of interest rates. The body of literature examined shows that several metrics are used to evaluate the performance of forecasting models. In line with widespread, supported practices we choose to evaluate our forecast by calculating the root mean square errors (RMSEs) and mean absolute errors (MAEs) of the residuals between our curve forecasts and actual data. We calculate the MAEs and RMSEs for each tenor and compared the calculated errors with the traditional benchmark of a random walk forecast (expectation hypothesis forecast) and a benchmark often used in institutional settings, the forward rates forecast (rational expectation forecast). In-sample and pseudo out-of sample forecasts are performed for all maturities and various forecasting horizons.

4.1 In-sample forecast

The in-sample forecast uses the entire available data sample until time t to estimate a fixed coefficient matrix B_1 of equation (4) which determines the dynamic behavior of the model parameters β_{i_t}. The corresponding path of NS parameters $\beta_{0_{t+h}}$, $\beta_{1_{t+h}}$ and $\beta_{2_{t+h}}$, a subset of β_i, are used to calculate the h-period ahead curve forecasts according to (2).
Tables 2 and 3 summarize the results obtained for all maturities.[7] The last two columns show the average MAE and RMSE. The calculated MAEs and RMSEs highlight that the in-sample forecast of our model (ExtNS) clearly outperforms forecasts based on forwards (Fwd) and a random walk (constant expectation). Despite the fact, that the in-sample forecast uses future data to evaluate the coefficient matrix and therefore can not represent a real-time forecast, it supports our forecasting model specification. A forecast performed today uses exactly the same coefficient matrix as the in-sample forecast. Assuming that a constant relationship between the variables of the VAR model takes a long time to converge, which is especially true for the heterogeneous economic environment observed throughout our data sample, in-sample forecasts could be an important guide to evaluating our forecasting performance.

[7]Despite the fact that MAEs and RMSEs are calculated for $h = 1$ to 36 months we only show results for $h = 12$ since the normative ranking of the evaluated models does not change.

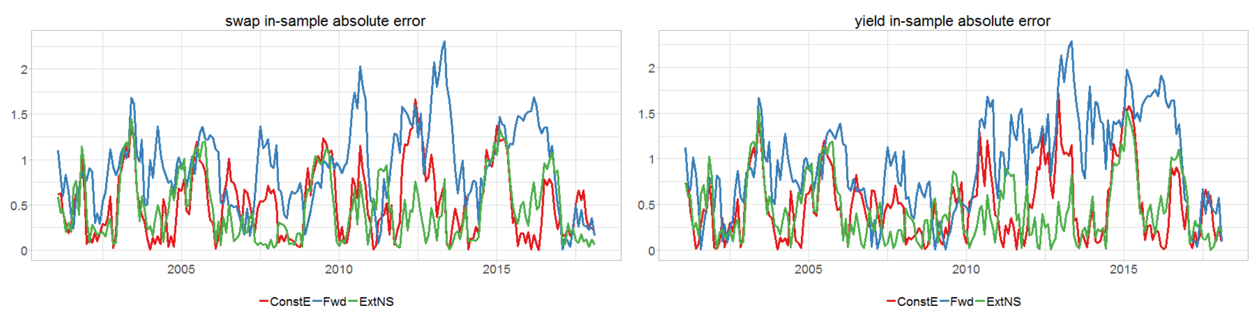
Table 2: 12-step ahead in-sample forecast errors for EA swap rates

	0.25	0.5	1	2	5	10	20	30	Mean RMSE	MAE
ExtNS	0.71	0.70	0.68	0.61	0.55	0.46	0.44	0.55	0.64	0.59
ConstE	0.70	0.69	0.66	0.62	0.58	0.52	0.51	0.51	0.67	0.60
Fwd	1.30	1.31	1.29	1.19	1.05	0.92	0.80	0.67	1.15	1.07

Table 3: 12-step ahead in-sample forecast errors for AT yields

	0.25	0.5	1	2	5	10	20	30	Mean RMSE	MAE
ExtNS	0.60	0.67	0.65	0.62	0.52	0.44	0.42	0.47	0.61	0.56
ConstE	0.65	0.64	0.63	0.62	0.57	0.49	0.48	0.47	0.65	0.57
Fwd	1.12	1.14	1.16	1.20	1.10	0.95	0.91	0.84	1.16	1.05

Figure 6: 12-step ahead in-sample forecast errors



4.2 Out-of sample

Unlike the in-sample forecast, the out-of-sample forecast only uses data available until forecasting point N to estimate the coefficient matrix B_1 of equation (4). With each additional data point that materializes, B_1 is re-evaluated. To capture the fact that the coefficient matrix B_1 needs a certain amount of observations (training period tr) to converge to stable values, out-of-sample forecasts can only be evaluated for $t > tr$. For our given data series B_1 converges after three years.[8]. Table 4 and 5 summarize the results obtained for all maturities. The calculated MAEs and RMSEs highlight that the out-of-sample forecast of our model (ExtNS) outperforms the forward forecast (Fwd), while the constant expectation forecast (ConstE) outperforms our model. The fact that constant expectation forecasts have a strong out-of-sample forecasting performance is already well known throughout interest rate forecasting literature. Our average results are therefore in line with alternative forecasting approaches.

Table 4: 12-step ahead out-of-sample forecast errors for EA swap rates

	0.25	0.5	1	2	5	10	20	30	Mean RMSE	MAE
ExtNS	1.01	1.00	0.98	0.91	0.82	0.66	0.59	0.69	0.90	0.83
ConstE	0.68	0.68	0.66	0.62	0.60	0.54	0.53	0.53	0.68	0.60
Fwd	1.30	1.32	1.31	1.20	1.07	0.95	0.82	0.71	1.17	1.09

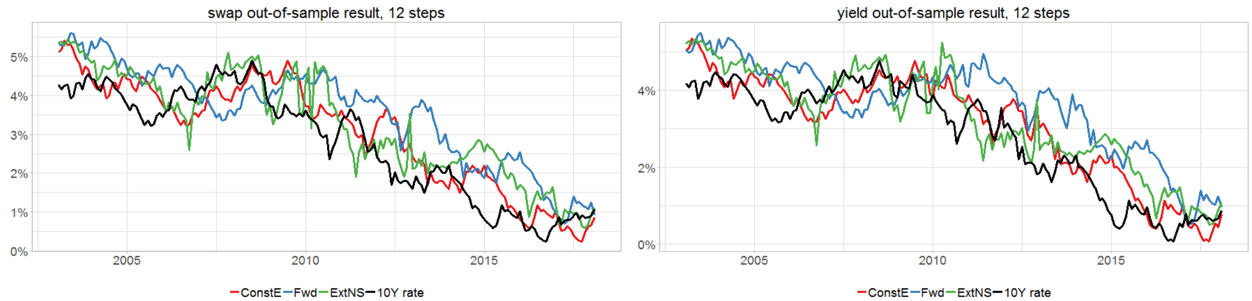
Table 5: 12-step ahead out-of-sample forecast errors for AT yields

	0.25	0.5	1	2	5	10	20	30	Mean RMSE	MAE
ExtNS	1.00	0.98	0.96	0.93	0.80	0.65	0.63	0.66	0.90	0.83
ConstE	0.64	0.63	0.63	0.62	0.58	0.51	0.51	0.49	0.66	0.58
Fwd	1.12	1.14	1.17	1.22	1.14	1.00	0.97	0.86	1.19	1.08

Many authors refer to the pseudo out-of-sample forecast evaluation as the "gold standard" of evaluation, since future data is simply treated as an unknown. Nevertheless, Clements and Hendry (2003) point out that this view should not be considered as a general rule.

[8]The 12-month ahead forecast presented in table (4) and (5) and figure (7) therefore only evaluates forecasts from 2002-01 onwards

Figure 7: 12-step ahead out-of-sample forecast errors



In our opinion, our framework, which sees a substantial heterogeneous time horizon (pre-crisis/crisis/post-crisis) with historically unobserved monetary policy reactions evaluated, offers an example of whether the sole consideration of out-of sample forecasting performance is justified. Clearly, in the light of potentially different policy regimes the out-of-sample residuals of our model are large, since it takes time for the coefficient matrix of the VAR to adjust/converge to new environments/values. The residuals are therefore partly attributed to the inability of our model to forecast changing policy environments or regimes. . Potential misleading interpretations derived from out-of-sample forecasting are pointed out by Duffee (2013), who indicates the importance of considering time horizons. As the in-sample performance of our model indicates, taking into account a constant coefficient matrix derived from a training period that considers different regimes substantially improves the overall forecasting performance compared with the constant expectation forecast. This is supported by the analysis performed in the following section where the out-of-sample forecast evaluation uses two separate time horizons representing different policy regimes.

5 Model extension: regime switch

Regime-dependent coefficient matrices B_1 and parameters β_i clearly have the potential to improve the forecast performance of a term-structure model (e.g.Bolder, 2002 and Bolder and Liu, 2007). The economic literature identifies two potential approaches to including different policy regimes in our framework. In Bolder (2002) a hidden-Markov chain model Hamilton (1989) is used to describe the economic business cycle which switches between economic expansion and recession, while Bolder and Liu (2007) exogenously impose different economic regimes onto the VAR system, identified by a recession-expansion dummy indicator (Demers, 2003). IN terms of the data sample taken into account in this paper the challenge of both approaches lies in determining ex-ante the future state of the economy or regime. In the case of the Markov-switching approach, the probability of staying in the current state is high. Due to the high regime persistence of this model class, expectations and forecasts change only marginally in the face of a potential regime switch and the marked shift in interest rates and yields following the 2008 financial crisis can not be generated. The second approach uses exogenous assumptions about future expansion-recession variables. Unfortunately, our data sample shows that, in particular, expectations about switches between expansions and recessions are highly uncertain. In our opinion, both approaches are unconvincing in their ability to identify the regime switches observed in our data sample. As a result, we do not use endogenous regime switches in our analysis.

To investigate our model's ability to forecast within a given policy regime we simply split our data sample into two periods: January 1999 to June 2012 ("pre-announcement") and July 2012 to January 2018 ("post-announcement").[9] These periods correspond to the time before and after the famous "what ever it takes" (July 26, 2012) statement by ECB President Mario Draghi which ushered in a new era of monetary policy making in Europe. Tables (6), (7), (8) and (9) show that the 12-month ahead relative in-sample and out-of-sample performance for the "pre-announcement" period improved substantially.

Table 6: In-sample result for EA swap rates until 2012-06

	0.25	0.5	1	2	5	10	20	30	Mean RMSE	MAE
ExtNS	0.84	0.84	0.81	0.72	0.57	0.47	0.45	0.59	0.72	0.66
ConstE	0.91	0.90	0.86	0.79	0.66	0.53	0.49	0.50	0.79	0.71
Fwd	1.50	1.50	1.45	1.30	1.06	0.88	0.76	0.65	1.24	1.14

Table 7: In-sample result for AT yields until 2012-06

	0.25	0.5	1	2	5	10	20	30	Mean RMSE	MAE
ExtNS	0.80	0.80	0.79	0.73	0.51	0.43	0.41	0.47	0.68	0.62
ConstE	0.85	0.84	0.82	0.77	0.60	0.45	0.43	0.42	0.74	0.65
Fwd	1.36	1.36	1.36	1.31	1.00	0.79	0.78	0.75	1.21	1.09

[9] Due to the short horizon of the post-announcement regime we shorten the training period for evaluating the coefficient matrix B_1 to 24 months

Table 8: Out-of-sample results for EA swap rates until 2012-06

	0.25	0.5	1	2	5	10	20	30	Mean RMSE	MAE
ExtNS	1.07	1.07	1.05	0.97	0.78	0.62	0.56	0.70	0.95	0.85
ConstE	0.94	0.93	0.89	0.83	0.70	0.56	0.52	0.54	0.83	0.74
Fwd	1.55	1.55	1.51	1.35	1.10	0.91	0.79	0.71	1.30	1.19

Table 9: Out-of-sample result for AT yields until 2012-06

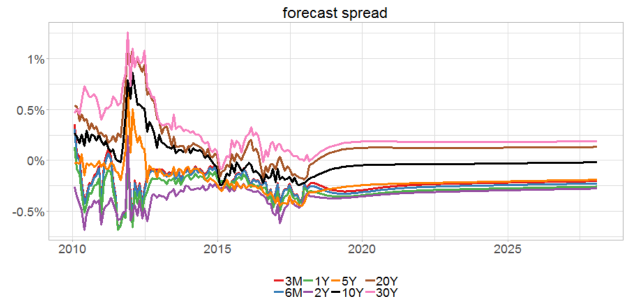
	0.25	0.5	1	2	5	10	20	30	Mean RMSE	MAE
ExtNS	1.09	1.08	1.06	0.99	0.73	0.59	0.58	0.60	0.94	0.84
ConstE	0.87	0.86	0.85	0.80	0.63	0.48	0.46	0.44	0.77	0.67
Fwd	1.41	1.41	1.41	1.35	1.04	0.84	0.85	0.76	1.27	1.13

6 Forecast until 2028

Using our derived extended NS model, we forecast the euro area term-structure of interest rates and the Austrian yield curve until 2028 (figure 8). The dynamics implicitly assume that the "new normal" is a combination of the pre- and post-financial crisis period where the interest rate factors and variables approach their historical average of 1999 to 2018 and 2002 to 2018 for euro area swaps and Austrian yields respectively. Our model forecasts almost continuously increasing swap rates and yields. Interestingly the spread between Austrian and euro area swap rates becomes smaller but stays negative over the entire time horizon for various tenors (e.g. 10-year yields, figure 9).

Figure 8: Yield and swap forecast until 2028

Figure 9: Spread forecast until 2028



7 Conclusion

Our study discusses the possibilities and merits of using a macro-financial euro area forecasting model to derive tools which enable the Austrian debt management office to perform risk management analyses and derive an optimal debt portfolio. Inspired by Diebold et al. (2006) and Bolder and Liu (2007) who use an extended macro-financial dynamic NS model for US and Canadian data, we applied the framework to European and Austrian swap and yield rate data. In line with the original literature, we consider the cyclical position of the economy (HP-filtered OECD industrial production data) and the HICP to be the relevant macroeconomic variables for describing term-structure dynamics. We perform our analysis for monthly European swap rate and Austrian yield rate data from 1999-01 to 2018-01. In-sample and out-of-sample forecasts are benchmarked against constant expectation forecasts (random walk) and forecasts employing the expectations hypothesis of the term-structure of interest rates (forward rate forecast). We find that the in-sample performance of our model clearly outperforms the forward rate and constant expectation forecast. As shown by various results throughout the literature (e.g. Molenaars et al., 2015 and den Butter and Jansen, 2013), the out-of-sample forecasting performance highlights the superiority of the constant expectation forecast. Nevertheless our model is again able to clearly outperform the forward rate forecast out-of-sample performance. When we take into account the existence of different policy regimes throughout our data sample the out-of-sample performance of our model improved substantially. Subsequently, our model forecasts the European term-structure of interest rates and the Austrian yield curve until 2028, deducing almost continuously increasing rates, with the 10-year EA swap rate and AT yield approaching 2.5% in 2028.

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